

# Ship Structural Optimization under Uncertainty

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## Abstract

*This article discusses the application of Genetic Algorithms to ship structural optimization and the treatment of variables with a degree of uncertainty. The variable uncertainty is included in the simulation using a Monte Carlo approach. The software ModeFrontier<sup>TM</sup> is applied in the case study for a double-hull tanker midship section design.*

## 1. Introduction

Optimization problems are problems in which one seeks to minimize or maximize a real-valued objective function by systematically choosing the values of real or integer variables from within an allowed set, which may be subject to constraints. Many real problems present uncertainty in their variables: They are inherent to the majority of physical, chemical, biological, geographical systems, etc. Stochastic optimization methods are optimization algorithms which incorporate probabilistic (random) elements, either in the problem data (the objective function, the constraints, etc.), or in the algorithm itself (through random parameter values, random choices, etc.), or in both. The concept contrasts with the deterministic optimization methods, where the values of the objective function are assumed to be exact, and the computation is completely determined by the values sampled so far. There are many stochastic optimization techniques. We will use here Genetic Algorithms (GAs) and Monte Carlo simulation.

Our objective is to minimize the section area of a double-hull tanker ship as a measure of weight. We use classification society based rules and investigate the possibility of uncertainty in some plate thickness. The main origin of the uncertainty could be the shipbuilding process and the corrosion behavior.

## 2. Methodology

### 2.1. Genetic algorithms

Genetic Algorithms are adaptive heuristic search algorithms built on the idea of genetics, natural selection and evolution. The basic concept of GAs is designed to simulate processes in natural system necessary for evolution, specifically those that follow the principles of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

The selective mechanisms carry out the changes that determine the evolution of a population over the generations. Such changes can occur due to the interactions between the individuals or due to the influence of the environment on the individual. Three basic mechanisms derive from this: crossover, reproduction and mutation. They are called genetic operators and are responsible for carrying out the evolution of the algorithm. The application of these operators is preceded by a selection process of the best adapted individuals, which uses a function called the fitness function (a.k.a. adaptation function).

An implementation of a genetic algorithm begins with a random population of chromosomes, i.e. the initial population can be obtained by choosing a value for the parameters or variables of each chromosome randomly between its minimum and maximum value. Then each individual is evaluated through the objective function. The fittest individuals (with the best adaptation values) have the greatest probability of reproducing (selection). Then genetic crossover and mutation operators work on the ones selected. The new individuals replace totally or partially the previous population, thus concluding a generation.

The selection operator allows the transmission of some individuals from the current population to the next one, with greater probability for the individuals with a better performance (fitness value), and with less probability for individuals with a worse performance. Crossover operators interchange and combine characteristics of the parents during the reproduction process, allowing the next generations to inherit these characteristics. The idea is that the new descendent individuals can be better than their parents if they inherit the best characteristics of each parent. The mutation operator is designed to introduce diversity into the chromosomes of the population of the GA, in order to ensure that the optimization process does not get trapped in local optima. In addition to these, there are other factors that influence the performance of a GA, adapted to the particularities of certain classes of problems.

## 2.2. Monte Carlo simulation

The present study uses the Monte Carlo simulation technique to deal with uncertainty concerning variables in optimization. The Monte Carlo method is a simulation technique used to solve probabilistic problems in which the input variables have probability distributions by means of a random process and obtaining as a result the distributions of probabilities of the output variables. The random process used consists in generating random numbers to select the values of each input variable for each attempt. This process is repeated many times, obtaining many results from which one builds a probability distribution of the output variables.

A random variable  $X$  has a normal distribution, with mean  $\mu$  ( $-\infty < \mu < +\infty$ ) and variance  $\sigma^2 > 0$ , if there is a density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

The notation  $X \sim N(\mu, \sigma^2)$  indicates that the random variable  $X$  has normal distribution with mean  $\mu$ , and variance  $\sigma^2$ .

The mean of the normal distribution is determined, making  $z = (x - \mu)/\sigma$  in the following equation:

$$E(X) = \int_{-\infty}^{\infty} \frac{x}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad (2)$$

$$E(X) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} (\mu + \sigma z) e^{-\frac{z^2}{2}} dz = \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz + \sigma \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} z e^{-\frac{z^2}{2}} dz$$

The normal density appears when integrating the first integral, with  $\mu = 0$  and  $\sigma^2 = 1$ , with this value being equal to one. The second integral has a zero value.

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} z e^{-\frac{z^2}{2}} dz = -\frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \Big|_{-\infty}^{\infty} = 0$$

$$E(X) = \mu[1] + \sigma[0] = \mu \quad (3)$$

The variance is determined, making  $z = (x - \mu)/\sigma$  in the following equations:

$$V(X) = E[(X - \mu)^2] = \int_{-\infty}^{\infty} (x - \mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{-\infty}^{\infty} \sigma^2 z^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \sigma^2 \int_{-\infty}^{\infty} \frac{z^2}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= \sigma^2 \left[ \frac{-ze^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \right] = \sigma^2 [0+1]$$

$$V(X) = \sigma^2 \tag{4}$$

Applying in the density equation an average equal to zero and variance 1, and making  $Z \sim N(0,1)$ , one has a standardized normal distribution.

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty \tag{5}$$

The corresponding distributed function is given by:

$$\phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du \tag{6}$$

And one can say that:

$$F(x) = \phi\left(\frac{x-\mu}{\sigma}\right) = \phi(z) \quad \text{with } Z = \frac{X-\mu}{\sigma} \tag{7}$$

For example, suppose that  $X \sim N(100,4)$  and we wish to find the probability of  $X$  being less than or equal to 104; that is to say,  $P(X \leq 104) = F(104)$ . Standardizing the point of interest  $x = 104$ , we obtain:

$$z = \frac{x-\mu}{\sigma} = \frac{104-100}{2} = 2$$

$$F(104) = \phi(2) = 0.9772$$

Thus the probability of the original normal random variable  $X$ , being less or equal to 104, is equal to the probability of the standardized normal random variable being less or equal to 2. There are tables where we can find accumulated standardized normal probability values for various values of  $z$ .

### 3. Double Hull Tanker Midship Section

The 1978 MARPOL Protocol introduced the concept known as protective location of segregated ballast tanks. The ballast tanks (which are empty on the cargo-carrying leg of the voyage and only loaded with water ballast for the return leg) are positioned where the impact of a collision or grounding is likely to be greatest. Thus the amount of cargo spilled after such an accident will be greatly reduced. The 1983 MARPOL amendments ban the carriage of oil in the forepeak tank - the ship's most vulnerable point in the event of a collision.

In 1992 MARPOL was amended to make it mandatory for tankers of 5000 dwt and more ordered after 6 July 1993 to be fitted with double hulls, or an alternative design approved by IMO (Regulation 13F (regulation 19 in the revised Annex I which entered into force on 1 January 2007) in Annex I of MARPOL 73/78). The requirement for double hulls that applies to new tankers has also been applied to existing ships under a program that began in 1995 (Regulation 13G (regulation 20 in the revised Annex I which entered into force on 1 January 2007) in Annex I of MARPOL 73/78). All tankers would have to be converted (or taken out of service) when reaching a certain age (up to 30 years old). This measure was adopted to be phased in over a number of years because shipyard capacity is limited and it would not be possible to convert all single hulled tankers to double hulls without causing immense disruption to world trade and industry.

This paper uses as case study the midship section design of a double-hull tanker to highlight the optimization procedure considering uncertainty in some variables. Optimization can be applied in many design phases, including preliminary design and detailed design. We focus here on the structural design phase, in particular the section modulus optimization under uncertainty. We assume the ship as beam. The section modulus is directly associated with the beam strength and the geometric material distribution.

$$SM = I / y \quad (8)$$

$I [m^4]$  denotes the moment of inertia and  $y [m]$  the distance from the neutral axis. The minimum and required SM following ABS (American Bureau of Shipping) rules are established in part 3, chapter 2, section 1 item 3.7.1 b as follows:

$$SM = C_1 C_2 L^2 B (C_b + 0.7) \quad (9)$$

$$C_1 = 10.75 - \left( \frac{300 - L}{100} \right)^{1.5} \quad \text{for } 90 \leq L \leq 300 \text{ m}$$

$$C_2 = 0.01$$

The required SM is calculated by:

$$SM = \frac{M_t}{f_p} = \frac{M_{sw} + M_w}{f_p} \quad (10)$$

$M_t$  denotes the total bending moment, composed of the calm-water moment  $M_{sw}$  and the wave moment  $M_w$ .  $f_p = 17.5 \text{ kN/cm}^2$ . The wave moment is calculated by:

$$\begin{aligned} M_{ws} &= -k_1 C_1 L^2 B (C_b + 0.7) \times 10^{-3} \text{ Sagging Moment} \\ M_{wh} &= +k_2 C_1 L^2 B C_b \times 10^{-3} \text{ Hogging Moment} \end{aligned} \quad (11)$$

With  $k_1 = 110$  and  $k_2 = 190$ .

We must also calculate the calm-water moment in hogging and sagging condition. We use the DNV (Det Norske Veritas) rules to calculate the  $M_{sw}$  moment:

$$\begin{aligned} M_{sw} &= -0.065 C_{wu} L^2 B (C_b + 0.7) \text{ (KNm) in sagging} \\ M_{sw} &= C_{wu} L^2 B (0.1225 - 0.015 C_b) \text{ (KNm) in hogging} \end{aligned} \quad (12)$$

$$C_{wu} = C_w \text{ for unrestricted service, with } C_w = \begin{array}{ll} 0.0792 L & \text{for } L \leq 100 \\ 10.75 - [(300-L)/100]^{3/2} & \text{for } 100 < L < 300 \\ 10.75 & \text{for } 300 \leq L \leq 350 \\ 10.75 - [(L-350)/150]^{3/2} & \text{for } L > 350 \end{array}$$

#### 4. Case Study

To illustrate the application of optimization under uncertainty techniques, we select a ship section modulus calculation. This normally is done in a spreadsheet. Fig. 1 presents the section used as case study. The main elements selected to optimize in this case study were: bottom, double bottom, deck side hull, side and bilge thickens plates.

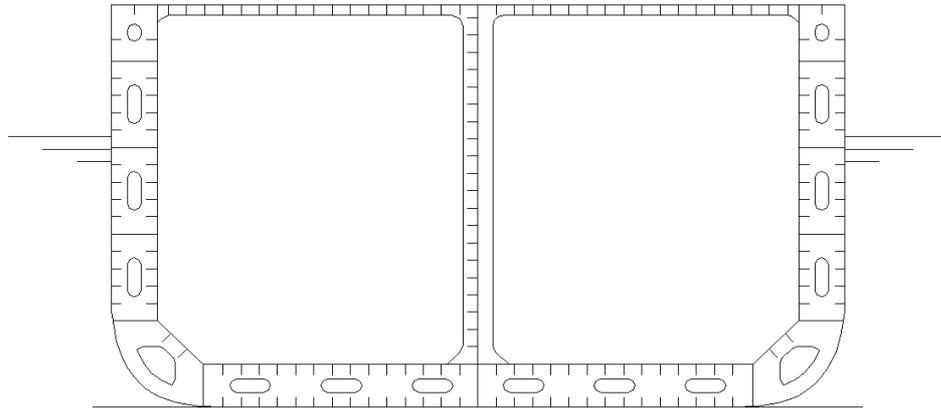


Fig. 1: Double Hull Tanker Midship Section

The optimization algorithm used is a single and multi objective simulated annealing (MOSA) algorithm. The main features are:

- a) Obeys boundary constraints on continuous variables
- b) Allows user defined discretization (base)
- c) Enforces user defined constraints by objective function penalization
- d) Allows concurrent evaluation of the n independent points

Fig. 2 presents the ModeFrontier™ model showing the variables, objectives and the mathematical model built in Excell™.

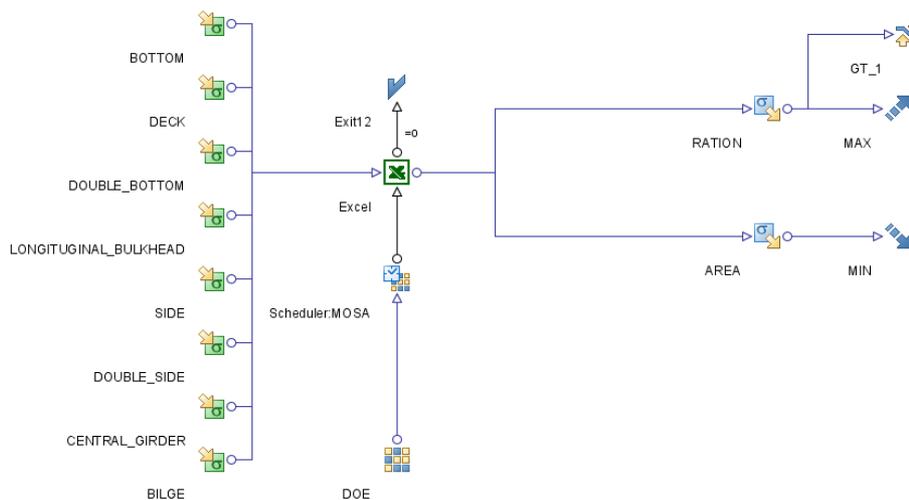


Fig. 2: ModeFrontier™ optimization model

The mathematical model includes eight variables and two objectives. The cost objective is represented by area (area of material or sectional material area) and the second objective represents the ratio between the calculated section modulus and the required section modulus. This indicates a safety aspect. Uncertainty was included in the eight variables with normal distribution with standard deviation of 5% in each thickness plate. The mathematical model is written in the Excell™ spreadsheet.

Genetic Algorithms are always time consuming and it was not different in this case. The model ran in a computer with 2 processors Quad-core (2.27 GHz) and took 3 hours and 10 minutes to calculate 2250 feasible models (in robust approach – which runs 10 times in uncertainty procedure) and

indicates the Pareto frontier with 42 designs. Fig. 3 shows the Pareto frontier for mean inertia and mean area ratio. Fig. 4 shows the bottom and deck thickness relation for all feasible designs. The selected designs in the Pareto frontier are indicated in green.

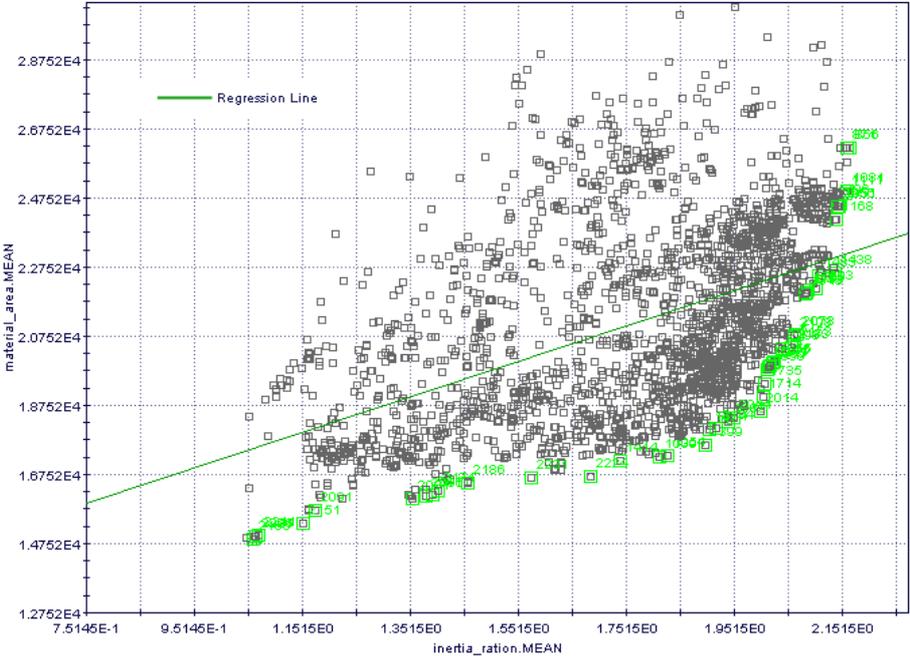


Fig 3: Mean Inertia and mean area Pareto frontier

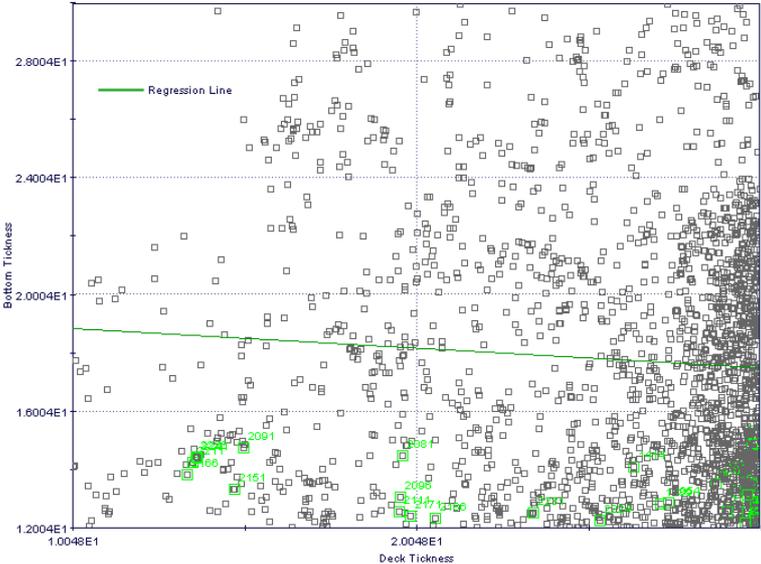


Fig 4: Mean Deck thickness and mean bottom thickness

Deb (2001) highlights the necessity to find a unique solution in almost all engineering and practical problems. The Pareto frontier was extracted from the feasible designs, Fig. 5. As a typical multi-objective problem, the increase in safety (mean ratio) means an increase in mean material area. The decision maker must select the best design and other aspects should also be considered. Deb (2001) called this phase as “Choose one of the obtained solutions using higher-level qualitative information”.

To illustrate the final process, Fig. 5 highlights four designs. Design 1 shows the lowest ratio and also the minimum mean area. This can be established as the most “optimistic” design. Design 4 in the other hand presents the highest safety aspect but also the most expensive. Design 2 is a good choice with 50% of “safety factor” and a very low mean area.

Design 3 with 5% more area (17000 to 18000) increases the “safety factor” from 50% to 100%. Also increasing the area after the point indicated by design 3 does not indicate a substantial gain in the “safety factor”. Table I presents the main elements of designs 2 and 3.

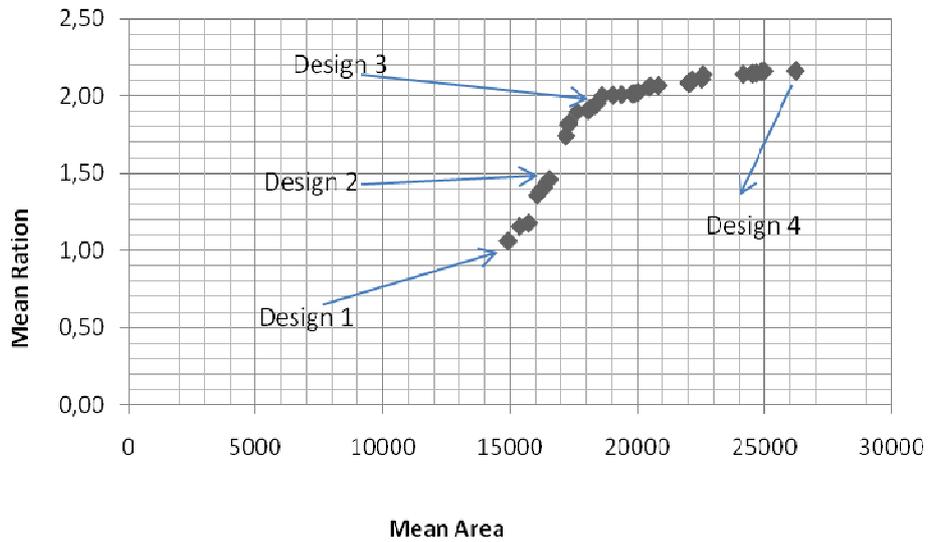


Fig 5: Feasible designs in Pareto Frontier

Table I: Main Elements with mean, maximum and minimum values

<b>DESIGN ID (42 Feasible Designs in Pareto Frontier)</b>	<b>2186</b>	<b>2014</b>
Long. Bulkhead Max	13	12
<b>Long. Bulkhead Mean</b>	<b>12</b>	<b>11</b>
Long. Bulkhead Min	11	10
Long. Bulkhead STDEV	1	0
Bilge.Max	15	13
<b>Bilge.Mean</b>	<b>14</b>	<b>11</b>
Bilge.Min	13	10
Bilge.STDEV	1	1
Deck Plate Max	22	32
<b>Deck Plate Mean</b>	<b>20</b>	<b>30</b>
Deck Plate Min	18	26
Deck Plate STDEV	1	2
Bottom Plate Max	13	13
<b>Bottom Plate Mean</b>	<b>12</b>	<b>12</b>
Bottom Plate Min	11	11
Bottom Plate STDEV	1	1
Double Bottom Plate Max	5	6
<b>Double Bottom Plate Mean</b>	<b>5</b>	<b>6</b>
Double Bottom Plate Min	4	6
Double Bottom Plate STDEV	0	0
Side Plate Max	14	17
<b>Side Plate Mean</b>	<b>12</b>	<b>16</b>
Side Plate Min	11	14
Side Plate STDEV	1	1
Double Side Plate Max	10	11
<b>Double Side Plate Mean</b>	<b>10</b>	<b>10</b>
Double Side Plate Min	9	9
Double Side Plate STDEV	0	1

Centre Girder.Max	29	28
<b>Centre Girder.Mean</b>	<b>27</b>	<b>26</b>
Centre Girder.Min	25	24
Centre Girder.STDV	1	1
area.Max	16736	18912
<b>area.Mean</b>	<b>16534</b>	<b>18617</b>
area.Min	16034	18136
area.STDEV	242	309
ratio.Max	1,53	2,12
<b>ratio.Mean</b>	<b>1,46</b>	<b>2,00</b>
ratio.Min	1,34	1,77
ratio.STDEV	0,06	0,26

## 5. Conclusion

The method presented here indicates the possibility to handle optimization problems where uncertainty should be attributed to any variable. The Monte Carlo method associated with genetic algorithms, as optimization procedures, worked well to solve the mathematical model with uncertainty.

The problem we faced was the time consumed using a common computer with duo core processor. The application for a section modulus calculation although simple, highlights the main methodology appliance. Naval Architecture designer can expect an interval of confidence for his section modulus and also evaluate the hull resistance characteristic in a risk approach.

## References

DEB, K. (2001), *Multi-Objective Optimization using Evolutionary Algorithms*, Wiley-Interscience Series in Systems and Optimization.